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ABSTRACT

Middle school algebra classes were observed to examine the multiple variables interacting in learning of algebra. Middle school algebra classes were especially informative because of the significant impact of the students' cognitive development transitions from concrete to formal operations. The National Council of Teachers of Mathematics (1989) informed much of the instructional practices in those classes observed. In contrast to much of the literature (i.e., Dickey, 1997), these teachers were effectively implementing developmentally appropriate and challenging algebra instruction. However, teachers acknowledged some challenges in meeting some of the NCTM standards such as use of real-life examples. Some suggestions were offered in this area as well as some recommendations to enhance metacognition in the middle school classes. (Contains 13 references.) (Author)

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Middle School Algebra: Variables for Success

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Abstract

Middle school algebra classes were observed to examine the multiple variables interacting in learning of algebra. Middle school algebra classes were especially informative because of the significant impact of the students' cognitive development transitions from concrete to formal operations. The National Council of Teachers of Mathematics (1989) informed much of the instructional practices in those classes observed. In contrast to much of the literature (i.e. Dickey (1997)), these teachers were effectively implementing developmentally appropriate and challenging algebra instruction. However, teachers acknowledged some challenges in meeting some of the NCTM standards such as use of real-life examples. Some suggestions were offered in this area as well as some recommendations to enhance metacognition in the middle school classes.

Middle School Algebra:

Variables for Success

Introduction

Middle school algebra classes are rich resources for analysis of the multiple dimensions inherent in learning mathematics. Key variables are the interrelationships between the content, cognitive development levels of the students, teachers' instructional approaches and related philosophies of math as a discipline. These connections are significant in implementation of the revised curriculum proposed by the National Council of Teachers of Mathematics (1989) which promoted major changes in standards for school mathematics. The NCTM standards' central tenets promoted understanding instead of rote memorization, active learning, problem-solving, as well as incorporation of technology into the mathematics curriculum.

Concrete and Formal Operational Thought

According to Jean Piaget (1964), typical middle school algebra students are in the cognitive development transition from concrete to formal operational cognitive functioning. Concrete and formal operational thought are two qualitatively different

stages of cognitive processing. Environmental interventions, such as teaching strategies, cannot accelerate the cognitive change from concrete to formal operations unless students' have attained the requisite development. Without the cognitive abilities to grasp abstractions, the learner will translate the abstract information into concrete and highly specific terms. For example, the learner may be able to define and graph linear equations like those presented by the teacher. However, if the concept of linear equation has been understood in a concrete way, the learner will not be able to generalize and engage in problem-solving different from what has been understood in the highly specific, concrete manner.

Piaget's critical point for educators is that activity produces cognitive challenges to prepare learners to move forward in thinking processes when developmentally ready. NCTM standards advocate active teaching approaches and encourage understanding. Thus, it may be assumed that these would promote cognitive growth in those middle school students who are developmentally prepared for the transition from concrete to formal operational thinking.

Constructivism and Teaching

Constructivist epistemologies have influenced recent thinking on the teaching and learning of mathematics (i.e. Ernest 1996). Constructivism states that the learner must construct his/her own knowledge. Concepts cannot be simply transferred from teacher to learner. The learner must make connections between information and develop individual schemas.

Piaget (1964) indicates that if the learner can relate the concept to previous knowledge then it will be assimilated into an existing schema. If not, the learner will accommodate to create a new schema. This internalization of experiences or creation of connections is critical to cognitive growth.

What does constructivism imply about teaching mathematics? First, the teacher must make creating knowledge constructs a teaching focus. Teaching cannot be a mere explanation of how to do mathematics. Secondly, the teacher must incorporate strategies to evaluate whether connections have been made by the learner. Schoenfeld (1992) encourages use of metacognitive strategies where the learner reflects on connections being made

or used. Crawford (1997) suggests having the learner describe thought processes orally or in writing during problem-solving. Another suggestion is to have students recall and use individually constructed schema in similar problem-solving situations. The learner may approach the new problem-solving scenarios individually or in a collaborative setting with other learners.

Constructivism and Vygotsky

The constructivist perspective of the learner as an active participant in the learning process is consistent with Vygotsky (1986). His construct of the zone of proximal development enriches understanding of the relationship between instruction and learning. The zone of proximal development can be described as the gap between the learner's level of actual development and potential development. Vygotsky proposes that the learner can solve more complex problems through collaboration and support of an adult or more capable peers than when working independently.

Vygotsky enriches the constructivist view of teaching because he sees the teacher's role as a mediator in the learning process.

Griffin and Cole (1984) clarify the teacher's instructional role with a learner in the zone of proximal development. The teacher and learner engage in social interactions where support is given to increase awareness and understanding. The extent of teacher support is flexible and based on feedback from the learner. For example, in an algebra class the teacher may ask for similarities and differences between two types of equations. If the learner can offer no response, then the teacher may suggest brainstorming any observable differences or may model "thinking aloud" to identify the similarities and differences.

What is Mathematics?

Vygotsky's description of instruction and learning as a process of social interactions is inherent in Schoenfeld (1992) and his discussion of mathematics instruction.. Schoenfeld conceptualizes mathematics as "an inherently social activity". Though Schoenfeld states abstraction, symbolic representation and manipulation are tools of the discipline, he sees mathematics as much more than training in the use of these tools. Mathematics must be viewed as a process where a learner develops a mathematical point of view and sees the

world as a mathematician would. Thus, the skills are acquired simultaneously with development of the point of view. This perspective justifies the need for mathematical instruction to be a social process. Through this process, the teacher assists the learner in construction of mathematical knowledge. Communication using mathematical terms, problem solving with real world examples, oral and written presentations, and collaborative work are a few strategies for promoting development of the learner's mathematical skills and point of view.

Present Study

In the present qualitative study, middle school algebra classes were selected as sites to observe the interplay of cognitive development, constructivism, and algebra as subject matter. The teacher and specific instructional strategies were clearly linked to these issues. Initially, students' levels of cognitive development were hypothesized to be the most dominant variable. While cognitive development was a critical variable, it was quickly discovered that many other factors were involved in the process of learning algebra in these classrooms.

Methods

Participants

Three middle school algebra classes were observed for this study in two different middle schools. The schools were part of the same local school system. Three teachers volunteered to participate in the study. Their teaching experiences ranged from one to seventeen years. One teacher had a degree in elementary education and a masters in administration, another had a degree in math and a masters in teaching. The third had a major in history and minors in math and secondary education. Students in the study were seventh graders enrolled in middle school algebra. The algebra classes averaged sixteen students per class.

Algebra classes from the same school system were selected to lend some control to the sample for this observational study. For example, students would be placed in middle school algebra classes using the same general criteria. Teachers reported that the minimal prerequisites were successful experiences in pre-algebra and solid foundations in arithmetic skills such as multiplication and adding of integers. Each of the observed

classes would be taught in a block scheduling format. The algebra text as well as classroom resources would be the same for all classes.

Design and Procedure

Qualitative research methods provide a large degree of freedom of inquiry. Thus, qualitative methods demand rigor in gathering, analyzing, and interpreting information. The present study included twelve hours of observations in algebra classes with observations of teacher, students, and content/instruction recorded every ten minutes. Since there was only one observer, all student activity could not be observed for each ten minute entry. The twelve hours of observation were divided into ninety-five minute blocks for each class observed.

Other sources of information included copies of the materials used by teachers and students. Observations of classroom environments and general resources in each were also recorded. Finally, the three teachers were interviewed individually as well as in a group. Their responses were recorded in writing by the observer. Immediately following the conclusion of each class observation and each interview, the information was reviewed

and editorial comments were added. All observations and interviews were done at mid-year of the middle schools' academic calendars.

Analysis of the data

Observations of instruction. In each of the algebra classes, the onset of class was structured with a review activity and highlights of topics to be covered that day. The review was usually presented visually and orally by the teachers. Reviews took many forms. These included: oral review using graphs on an overhead, review of graded quizzes, checking homework, or worksheets to be completed in class. The teachers encouraged student engagement in this review by asking specific students for responses, demonstrating a process and allowing students to explain how and why, promoting peer collaboration in finding solutions, or class and teacher "testing" a student's idea for an answer to a problem.

In interviews, the teachers advocated the need for this review phase because it helped create connections for the students between previous knowledge and the new concepts. The teachers also thought that the review was essential because it

helped insure transfer to long term memory. Due to the block schedule, the middle school students did not meet every day so this was another benefit of the review.

These algebra teachers liked the block schedule because it allowed time for thorough review and then the introduction of new materials. These middle school teachers contrasted with findings from Kramer (1997) who found that math teachers generally had great difficulties in adapting teaching strategies to fit block schedules. Two of the teachers had also taught daily fifty-minute classes and felt that those limited the time allotted for review. The limitations were in part because of the need to cover new information in each class. Thus, review could be given only a minimal focus.

Teachers played critical roles as facilitators (Griffin and Cole, 1994) to bridge the students' transitions from concrete to formal operational thinking. First, the review was always incorporated into the lessons to assist in creating links between old and new information. New concepts were presented in concrete ways that then progressed to the more abstract concept being taught. Teachers routinely incorporated questions

to help the cognitive transition to more complex concepts. The questions generally offered clues or connections between concrete and abstract thought. For example, the teacher might ask what would be the next step or what was similar to an earlier problem.

Observations of cognitive development. The concrete to abstract progression was important to the middle school students' learning, especially based on observations of their cognitive development levels. For example, the teacher showed a linear graph and asked what it represented. The student replied that it was .."a line that was able to move". The teacher then clarified by saying that this line indicated all the possible answers to the problem. In another class, a student could not understand why she was unable to solve an equation. Suddenly, she had an "oh, oh now I know" response, when the teacher concretely explained that the variables must look alike to combine them. During this same class, another student needed to repeat for himself ..."can't add a's and b's" as he independently completed an assignment.

Other issues relevant to cognitive transitions from concrete to abstract reasoning included the students' need to have frequent verbal and visual clarification that two variable equations could have more than one answer. The teacher needed to demonstrate the differences between one and two variable equations and then have students explain. Students still had to say aloud that there would be more than one possible answer as they began to attempt solutions to two variable equations. In this lesson, students also were encouraged to work together to find solutions. During instruction, teachers engaged all students through questions because some adolescents might be reluctant to ask if they did not understand for fear of peer disapproval. Not only were these middle school algebra teachers sensitive to cognitive development needs of their students, they were also keenly aware of social and physical development. Students were allowed to work in groups and converse quietly at selected times in two of the teachers' classes. Since the block schedule class was approximately an hour and a half long, teachers incorporated activities that had students move around or a "stretch break" was given midway through the class. One class

met just before noon and the teacher let the students bring a snack to be eaten at the beginning of class. This routine did not interfere with classtime because the students were engaged in getting materials for review at the same time.

NCTM Standards. NCTM recommendations for mathematics were evident in the observations of instruction, materials, and teacher philosophies of math. The teachers promoted active learning through group problem-solving, frequent discussions of problems and solutions, and use of real-life examples. For example, weight loss and saving money were presented as applications of linear equations.

Understanding and constructing knowledge were evident because teachers encouraged students to explore, not merely find solutions. For example, graphing calculators were introduced with a few simple directions. Then students were given guided practice to examine the technology and discover some of its capabilities. The rationale for use of graphing calculators to solve linear equations was also presented to connect the technology to previously presented concepts.

NCTM curriculum standards for algebra in middle school stated that emphasis should be on informal exploration with data, graphs, and models. The students should be able to generalize and use numbers to represent problems and observed physical patterns. In all the middle school classes observed, these strategies were used in instruction to help students bridge the concrete to abstract thought processes. Limited new information was introduced before extensive practice was given. The practice was in multiple formats such as whole-class solutions to problems, discussions of how the new concepts related to previous knowledge, or individual guided practice activities.

The NCTM standards also stated that algebra students should understand the concept of variable and solve linear equations. However, the standards advised that it would be especially important for metacognitive strategies to be used with middle school students to check for formal understanding of mathematical ideas. In other words, it could not be assumed that middle school students would readily grasp an abstract concept from a mathematical activity. For example, the concept

of variable might be interpreted as only a letter representing a number rather than interpretation based on how the variable was used in context. In one of the algebra classes observed, several students were confused about two variable equations and demonstrated the lack of formal comprehension of the concept of variable. The students still tried to solve for one correct solution to the two variable equations. This again highlighted concrete cognitive processing and the need for the teachers to adhere to professional practices of collaboration, reasoning, and even models to assist in the transition to more abstract understanding of the concept of variable.

Constructivism in the classroom. The constructivist approach to learning algebra was a clear teaching focus in these algebra classes. As Schoenfeld (1992) suggested, these teachers taught and understood algebra as a social process. Much verbal exchange occurred between teacher and students as well as between students in groups. Math vocabulary was taught directly with definitions. Then, the vocabulary was used in context throughout the classes. The classroom environments had colorful posters demonstrating mathematical concepts. One

poster showed examples of Fiboracci numbers in nature. Another class, displayed students' algebra work on a bulletin board. Some worksheets and activities also used cartoon characters or student names to bring relevance to the algebra assignment.

Discussion

The middle school algebra classes were active teaching and learning environments. Instruction was designed to connect concrete and formal operational understanding of algebraic concepts. Teachers utilized NCTM guidelines for active learning, understanding, problem-solving, and use of relevant technology to create these connections. Metacognitive approaches were sometimes observed when students were asked to explain an answer or when students solved problems in a collaborative effort with peers. The reflections were always oral and generally solicited by questions from the teachers.

Students were beginning to achieve the NCTM standards for algebra in the middle school in these midyear observations. These included an initial understanding of variable and the abilities to use formal solutions to solve linear equations. However, at the time of observations these middle school

students were very limited in their abilities to generalize their understanding. These abilities should develop with additional instruction and practice to bridge the gap between concrete and formal cognitive processing.

Recommendations. Based on this qualitative study, learning of algebra was occurring amidst an interplay of effective and appropriate instruction, relevant curriculum guidelines, and awareness of cognitive development issues. Several recommendations could be considered for enhancing the learning processes. First, oral and written communications would enrich the reflective processes for the middle school students. Using written expression, the teachers might have students explain their thought processes in solving problems to discover any fallacies or confusions. Secondly, models could be another teaching tool to connect concrete to abstract processing. Such models may include the "M and M Challenge" or the "Beads Under the Cloud" (Gorrell and Collins, 1994). These models required the discovery of patterns and use of peer collaboration. These may also enrich possibilities for developing generalizations.

The teachers in the study expressed their desires for more real-life examples to use in their instruction. Limited review of the literature found few relevant algebra examples for middle school students. One program, The Interactive Mathematics Program (1995), included several examples such as related processes to maximize profits for a bakery. Since this lack of realistic examples for problem-solving appeared to be a need within the discipline, it would be advantageous for middle school algebra teachers to collaborate and create a series of real-life examples for their classes.

Finally, word problems were not welcomed by the students in the classes observed. Two insights are offered on this concern. The first would be that students need more opportunities to construct their own mental models to convert language to math and vice versa. Pogrow (1994) used software where students had to write their own word problems to a space creature. The software was designed to give feedback to the students on the word problems they had created. Though software may not be readily available for such practice, analogous work could be developed to enhance the translation of math to language. A

second suggestion would be to incorporate relevant materials in the word problems. This again presents the challenge for real - life examples for middle school algebra. Newspapers and magazines may be useful for materials. Graphs, economic reports, advertisements for weight loss products would be possible resources to incorporate into word problems.

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